Lab 2: Approximating areas

1 Introduction

The purpose of this lab is to show how the spreadsheet can be used to find approximate areas under curves, and to convince you that the Fundamental Theorem of Calculus actually works.

Let’s assume \( f(t) \) is positive for \( a \leq t \leq b \). Then the Fundamental Theorem of Calculus says that the area under the curve \( y = f(t) \) between \( t = a \) and \( t = x \leq b \) is

\[
\int_a^x f(t) dt = F(x) - F(a),
\]

where \( F(t) \) can be any anti-derivative of \( f(t) \).

In the following problems you’ll be asked to compute the area under such a curve in three different ways:

- Precisely, by using an anti-derivative as above
- Approximately, by filling the area under the curve up with 20 thin rectangles and adding up their areas.
- Approximately again, but using 50 rectangles rather than 20.

You’ll then compare the answers you got and tell us the error in the approximations. The 50–rectangle approximation will be more accurate. You will also illustrate all of these steps using Mathsheet’s line graphs and bar graphs.
2 Problems

Note: This lab, and other labs contain several very similar sets of problems.

You only do ONE problem set.
Your student number is used to indicate which problem is assigned to you.

- Do Problem Set 1 if the last digit of your student number is a 1 or a 6
- Do Problem Set 2 if the last digit of your student number is a 2 or a 7
- Do Problem Set 3 if the last digit of your student number is a 3 or a 8
- Do Problem Set 4 if the last digit of your student number is a 4 or a 9
- Do Problem Set 5 if the last digit of your student number is a 5 or a 0

Note: Please label each graph carefully with your name, student number, lecture section, and indicate which problem set you did. Also, don’t forget to write the answers to all questions on the bottom of the page, directly under the graph.

If you are asked to hand in more than one page, please staple the pages.
2.1 Problem Set 1

Consider the function

\[ f(x) = \frac{5x^4}{4}. \]

Using the spreadsheet, create one plot showing parts (a)-(d) superimposed. Repeat this procedure for part (e).

(a) a graph of the function \( f \) over the domain \( 1.5 \leq x \leq 3 \). Do an \( xy \)-plot, not a bar graph.

(b) a bar graph showing 20 rectangular strips whose top right corners lie on the graph of the function. (Note: create this graph before part (a), so that it does not obscure the curve.)

(c) a graph of the function \( S(n) \) which adds up the areas the first \( n \) strips \( (S(1) \) is the area of the first strip, \( S(2) \) is the combined areas of the first two strips, \ldots\). Show this function as a curve from points \( (1.5, S(0) = 0) \) to \( (3, S(20)) \).

(d) Find an antiderivative \( A(x) \) of \( f(x) \), and draw a graph of \( A(x) - A(1.5) \) as a curve on the given interval. The Fundamental Theorem of Calculus says that \( A(x) - A(1.5) \) should be the area underneath the graph over the interval \([1.5, x]\).

(e) Repeat the same procedure using 50 rectangular strips to cover the same area. Determine the total area as calculated by the spreadsheet and compare it with the value predicted by the anti-derivative.

(f) Find the error in the approximate area under \( f(x) \) for \( 1.5 \leq x \leq 3 \) for parts (c) and (e).

You’ll have to hand in one page with...
• the printout of the graph produced for parts (a)–(d),
• the value $S(20)$ produced in (c) for the area of the bars, (c),
• the value of the area of the bars produced in (e),
• the formula for the antiderivative $A(x)$, and
• the value $A(3) - A(1.5)$. This should be the true area under the curve $y = 5x^4/4$ over the interval $[1.5, 3]$, according to the Fundamental Theorem.

• the values of the errors in the two approximations.

Please be sure that your axes are labeled, and that it is clear which curve is which (since some of them will be rather close together). You may write the formula for $A(x)$ and the results of the calculations beneath your graph.

Note that we don’t need you to hand in the second graph, with 50 bars on it, in order to save paper. However, we strongly recommend that you make the graph and inspect it, so that you can be sure that you are computing the values correctly. If all has gone well, the graph for part (d) should lie very close to the graph of part (c), but they will not be exactly the same (since one is an approximation of the other).
2.2 Problem Set 2

Consider the function

\[ f(x) = \frac{4x^2}{5}. \]

Using the spreadsheet, create one plot showing parts (a)-(d) superimposed. Repeat this procedure for part (e).

(a) a graph of the function \( f \) over the domain \( 0.5 \leq x \leq 3 \). Do an \( xy \)-plot, not a bar graph.

(b) a bar graph showing 20 rectangular strips whose top right corners lie on the graph of the function. (Note: create this graph before part (a), so that it does not obscure the curve.)

(c) a graph of the function \( S(n) \) which adds up the areas the first \( n \) strips \( (S(1) \) is the area of the first strip, \( S(2) \) is the combined areas of the first two strips, \ldots \). Show this function as a curve from points \( (0.5, S(0) = 0) \) to \( (3, S(20)) \).

(d) Find an antiderivative \( A(x) \) of \( f(x) \), and draw a graph of \( A(x) - A(0.5) \) as a curve on the given interval. The Fundamental Theorem of Calculus says that \( A(x) - A(0.5) \) should be the area underneath the graph over the interval \([0.5, x]\).

(e) Repeat the same procedure using 50 rectangular strips to cover the same area. Determine the total area as calculated by the spreadsheet and compare it with the value predicted by the anti-derivative.

(f) Find the error in the approximate area under \( f(x) \) for \( 0.5 \leq x \leq 3 \) for parts (c) and (e).

You’ll have to hand in one page with...
• the printout of the graph produced for parts (a)–(d),
• the value $S(20)$ produced in (c) for the area of the bars, (c),
• the value of the area of the bars produced in (e),
• the formula for the antiderivative $A(x)$, and
• the value $A(3) - A(0.5)$. This should be the true area under the curve $y = 4x^2/5$ over the interval $[0.5, 3]$, according to the Fundamental Theorem.
• the values of the errors in the two approximations.

Please be sure that your axes are labeled, and that it is clear which curve is which (since some of them will be rather close together). You may write the formula for $A(x)$ and the results of the calculations beneath your graph.

Note that we don’t need you to hand in the second graph, with 50 bars on it, in order to save paper. However, we strongly recommend that you make the graph and inspect it, so that you can be sure that you are computing the values correctly. If all has gone well, the graph for part (d) should lie very close to the graph of part (c), but they will not be exactly the same (since one is an approximation of the other).
2.3 Problem Set 3

Consider the function

\[ f(x) = \frac{5x^3}{8}. \]

Using the spreadsheet, create one plot showing parts (a)-(d) superimposed. Repeat this procedure for part (e).

(a) a graph of the function \( f \) over the domain \( 0.5 \leq x \leq 2 \). Do an \( xy \)-plot, not a bar graph.

(b) a bar graph showing 20 rectangular strips whose top right corners lie on the graph of the function. (Note: create this graph before part (a), so that it does not obscure the curve.)

(c) a graph of the function \( S(n) \) which adds up the areas the first \( n \) strips \( (S(1) \) is the area of the first strip, \( S(2) \) is the combined areas of the first two strips, \ldots). Show this function as a curve from points \( (0.5, S(0) = 0) \) to \( (2, S(20)) \).

(d) Find an antiderivative \( A(x) \) of \( f(x) \), and draw a graph of \( A(x) - A(0.5) \) as a curve on the given interval. The Fundamental Theorem of Calculus says that \( A(x) - A(0.5) \) should be the area underneath the graph over the interval \([0.5, x]\).

(e) Repeat the same procedure using 50 rectangular strips to cover the same area. Determine the total area as calculated by the spreadsheet and compare it with the value predicted by the anti-derivative.

(f) Find the error in the approximate area under \( f(x) \) for \( 0.5 \leq x \leq 2 \) for parts (c) and (e).

You’ll have to hand in one page with…(next page)
• the printout of the graph produced for parts (a)–(d),
• the value $S(20)$ produced in (c) for the area of the bars, (c),
• the value of the area of the bars produced in (e),
• the formula for the antiderivative $A(x)$, and
• the value $A(2) - A(0.5)$. This should be the true area under the curve $y = \frac{5x^3}{8}$ over the interval $[0.5, 2]$, according to the Fundamental Theorem.
• the values of the errors in the two approximations.

Please be sure that your axes are labeled, and that it is clear which curve is which (since some of them will be rather close together). You may write the formula for $A(x)$ and the results of the calculations beneath your graph.

Note that we don’t need you to hand in the second graph, with 50 bars on it, in order to save paper. However, we strongly recommend that you make the graph and inspect it, so that you can be sure that you are computing the values correctly. If all has gone well, the graph for part (d) should lie very close to the graph of part (c), but they will not be exactly the same (since one is an approximation of the other).
2.4 Problem Set 4

Consider the function $f(x) = \frac{5x^4}{2}$.

Using the spreadsheet, create one plot showing parts (a)-(d) superimposed. Repeat this procedure for part (e).

(a) a graph of the function $f$ over the domain $1.5 \leq x \leq 2$. Do an $xy$–plot, not a bar graph.

(b) a bar graph showing 20 rectangular strips whose top right corners lie on the graph of the function. (Note: create this graph before part (a), so that it does not obscure the curve.)

(c) a graph of the function $S(n)$ which adds up the areas the first $n$ strips ($S(1)$ is the area of the first strip, $S(2)$ is the combined areas of the first two strips, ...). Show this function as a curve from points $(1.5, S(0) = 0)$ to $(2, S(20))$.

(d) Find an antiderivative $A(x)$ of $f(x)$, and draw a graph of $A(x) - A(1.5)$ as a curve on the given interval. The Fundamental Theorem of Calculus says that $A(x) - A(1.5)$ should be the area underneath the graph over the interval $[1.5, x]$.

(e) Repeat the same procedure using 50 rectangular strips to cover the same area. Determine the total area as calculated by the spreadsheet and compare it with the value predicted by the anti-derivative.

(f) Find the error in the approximate area under $f(x)$ for $1.5 \leq x \leq 2$ for parts (c) and (e).

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• the value $A(2) - A(1.5)$. This should be the true area under the curve $y = \frac{5x^4}{2}$ over the interval $[1.5, 2]$, according to the Fundamental Theorem.
• the values of the errors in the two approximations.

Please be sure that your axes are labeled, and that it is clear which curve is which (since some of them will be rather close together). You may write the formula for $A(x)$ and the results of the calculations beneath your graph.

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2.5 Problem Set 5

Consider the function

\[ f(x) = \frac{x^3}{3}. \]

Using the spreadsheet, create one plot showing parts (a)-(d) superimposed. Repeat this procedure for part (e).

(a) a graph of the function \( f \) over the domain \( 1.5 \leq x \leq 2.5 \). Do an \( xy \)-plot, not a bar graph.

(b) a bar graph showing 20 rectangular strips whose top right corners lie on the graph of the function. (Note: create this graph before part (a), so that it does not obscure the curve.)

(c) a graph of the function \( S(n) \) which adds up the areas the first \( n \) strips \((S(1) \) is the area of the first strip, \( S(2) \) is the combined areas of the first two strips, \ldots\). Show this function as a curve from points \((1.5, S(0) = 0)\) to \((2.5, S(20))\).

(d) Find an antiderivative \( A(x) \) of \( f(x) \), and draw a graph of \( A(x) - A(1.5) \) as a curve on the given interval. The Fundamental Theorem of Calculus says that \( A(x) - A(1.5) \) should be the area underneath the graph over the interval \([1.5, x]\).

(e) Repeat the same procedure using 50 rectangular strips to cover the same area. Determine the total area as calculated by the spreadsheet and compare it with the value predicted by the anti-derivative.

(f) Find the error in the approximate area under \( f(x) \) for \( 1.5 \leq x \leq 2.5 \) for parts (c) and (e).

You’ll have to hand in one page with...
• the printout of the graph produced for parts (a)–(d),
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• the value $A(2.5) - A(1.5)$. This should be the true area under the curve $y = x^3/3$ over the interval $[1.5, 2.5]$, according to the Fundamental Theorem.
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