1 Introduction

A function $f$ of one variable $x$ describes a curve in the $xy$-plane in the following way: The curve consists of all points $(x, y)$ whose coordinates satisfy the equation

$$y = f(x).$$

The purpose of this lab is to compute the length of this curve using the arclength formula. You should read the course notes, especially chapter 5, to refresh your memory on the meaning of arclength and how it is calculated. Recall that the length of the curve $y = f(x)$ for $a \leq x \leq b$ is given by the arclength formula

$$\ell(x) = \int_a^b \frac{d\ell}{dx} \, dx$$

where

$$\frac{d\ell}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

We sometimes refer to $d\ell$ as “an element of arclength”, and to $\frac{d\ell}{dx}$ as the rate of change of arclength. Here we will investigate how the function $f$, the rate of change of arclength, and the total accumulated length depend on $x$ in the given interval.
2 Problems

Note: This lab, and other labs contain several very similar sets of problems.

You only do ONE problem set.
Your student number is used to indicate which problem is assigned to you.

- Do Problem Set 1 if the last digit of your student number is a 1 or a 6
- Do Problem Set 2 if the last digit of your student number is a 2 or a 7
- Do Problem Set 3 if the last digit of your student number is a 3 or a 8
- Do Problem Set 4 if the last digit of your student number is a 4 or a 9
- Do Problem Set 5 if the last digit of your student number is a 5 or a 0

Note: Please label each graph carefully with your name, student number, lecture section, and indicate which problem set you did. Also, don’t forget to write the answers to all questions on the bottom of the page, directly under the graph.

If you are asked to hand in more than one page, please staple the pages.
2.1 Problem Set 1

You are told that the derivative of a certain function $f$ is

$$f'(x) = 2x \cos(x^2)$$

and that the point $(0, 0)$ lies on the curve described by that function. Use the spreadsheet to create one page that contains the following graphs:

- The graph $y = f(x)$ of the function $f$ (whose derivative is given to you). This should be plotted over the interval from 0 to 2.

- The graph of the derivative of the "arclength function" $\ell$

  $$\frac{d\ell}{dx}(x) = \sqrt{1 + f'(x)^2},$$

  showing how it varies across the same interval.

- The graph of the approximation to the arclength $\ell(x)$ of the curve from 0 to $x$, for all values $x$ between 0 and 2.

You should use a very small step size to get good approximations to the exact solutions. If you see any sharp corners on your graph, then you must use a smaller step size. Make sure that the graph doesn’t change considerably when you decrease the step size.

**Surveys:** Please fill out the lab survey at surveys.
2.2 Problem Set 2

You are told that the derivative of a certain function \( f \) is

\[ f'(x) = 1.6x \sin(x^2) \]

and that the point \((0, 0)\) lies on the curve described by that function. Use the spreadsheet to create one page that contains the following graphs:

- The graph \( y = f(x) \) of the function \( f \) (whose derivative is given to you). This should be plotted over the interval from 0 to 2.

- The graph of the derivative of the "arclength function" \( \ell \)

\[
\frac{d\ell}{dx}(x) = \sqrt{1 + f'(x)^2},
\]

showing how it varies across the same interval.

- The graph of the approximation to the arclength \( \ell(x) \) of the curve from 0 to \( x \), for all values \( x \) between 0 and 2.

You should use a very small step size to get good approximations to the exact solutions. If you see any sharp corners on your graph, then you must use a smaller step size. Make sure that the graph doesn’t change considerably when you decrease the step size.

**Surveys:** Please fill out the lab survey at surveys.
2.3 Problem Set 3

You are told that the derivative of a certain function $f$ is

$$f'(x) = 1.6x \sin(x^2)$$

and that the point $(0, 0)$ lies on the curve described by that function. Use the spreadsheet to create one page that contains the following graphs:

- The graph $y = f(x)$ of the function $f$ (whose derivative is given to you). This should be plotted over the interval from 0 to 2.

- The graph of the derivative of the "arclength function" $\ell$

$$\frac{d\ell}{dx}(x) = \sqrt{1 + f'(x)^2},$$

showing how it varies across the same interval.

- The graph of the approximation to the arclength $\ell(x)$ of the curve from 0 to $x$, for all values $x$ between 0 and 2.

You should use a very small step size to get good approximations to the exact solutions. If you see any sharp corners on your graph, then you must use a smaller step size. Make sure that the graph doesn’t change considerably when you decrease the step size.

Surveys: Please fill out the lab survey at surveys.
2.4 Problem Set 4

You are told that the derivative of a certain function $f$ is

$$f'(x) = 1.8x \cos(x^2)$$

and that the point $(0, 0)$ lies on the curve described by that function. Use the spreadsheet to create one page that contains the following graphs:

- The graph $y = f(x)$ of the function $f$ (whose derivative is given to you). This should be plotted over the interval from 0 to 3.

- The graph of the derivative of the "arclength function" $\ell$

$$\frac{d\ell}{dx}(x) = \sqrt{1 + f'(x)^2},$$

showing how it varies across the same interval.

- The graph of the approximation to the arclength $\ell(x)$ of the curve from 0 to $x$, for all values $x$ between 0 and 3.

You should use a very small step size to get good approximations to the exact solutions. If you see any sharp corners on your graph, then you must use a smaller step size. Make sure that the graph doesn’t change considerably when you decrease the step size.

Surveys: Please fill out the lab survey at surveys.
2.5 Problem Set 5

You are told that the derivative of a certain function $f$ is

$$f'(x) = 1.9x \cos(x^2)$$

and that the point $(0, 0)$ lies on the curve described by that function. Use the spreadsheet to create one page that contains the following graphs:

- The graph $y = f(x)$ of the function $f$ (whose derivative is given to you). This should be plotted over the interval from 0 to 3.5.

- The graph of the derivative of the "arclength function" $\ell$

$$\frac{d\ell}{dx}(x) = \sqrt{1 + f'(x)^2},$$

showing how it varies across the same interval.

- The graph of the approximation to the arclength $\ell(x)$ of the curve from 0 to $x$, for all values $x$ between 0 and 3.5.

You should use a very small step size to get good approximations to the exact solutions. If you see any sharp corners on your graph, then you must use a smaller step size. Make sure that the graph doesn’t change considerably when you decrease the step size.

**Surveys:** Please fill out the lab survey at surveys.